MATH 134A Review: Eigenvalues and Eigenvectors

Facts to Know

An eigenvalue λ and corresponding eigenvector (in the λ-eigenspace) of an $n \times n$ matrix A is

(λ, v) $\qquad A v = \lambda v$

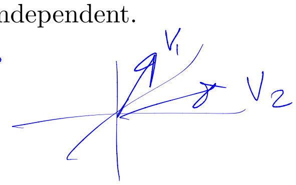
$(\lambda, 2v)$ $\qquad A(2v) = 2(Av) = 2(\lambda v) = \lambda(2v)$

Theorem: the eigenvalues of a triangular matrix are the entries on its main diagonal

$$\begin{bmatrix} 1 & 9 & 9 \\ 0 & 2 & 9 \\ 0 & 0 & 3 \end{bmatrix} \qquad \lambda = 1, 2, 3$$

Theorem: If $\mathbf{v}_1$ and $\mathbf{v}_2$ are two eigenvectors corresponding to distinct eigenvalues λ_1 and λ_2, then $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent.

$\mathbb{R}^n = \mathbb{R}^3$



A method to find the eigenvalues of a matrix is

General method $\qquad \det(A - \lambda I) = 0$

variable

Solve for λ

A method to find a basis for the eigenspace corresponding to a given eigenvalue is

$$(A - \lambda I) v = 0$$

Solve for v

these are precisely the λ-eigenvectors

Examples

1. Find one eigenvalue for the following matrix.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \qquad v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$Av = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} = 6\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= 6v$$

λ

2. Find a basis for the corresponding eigenspace.

$$A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}, \quad \lambda = 2$$

$Av = 2v$ Find all v

$(A - 2I)v = 0$

$$\left[\begin{array}{c|c} A - 2I & \vec{0} \end{array}\right] \qquad \left[\begin{array}{ccc|c} 2 & -1 & 6 & 0 \\ 2 & -1 & 6 & 0 \\ 2 & -1 & 6 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4-2 & -1 & 6 & 0 \\ 2 & 1-2 & 6 & 0 \\ 2 & -1 & 8-2 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 2 & -1 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$2x - s + 6t = 0$

$x = (-6t + s)/2$

Solution

$$\begin{bmatrix} (-6t+s)/2 \\ s \\ t \end{bmatrix} = \begin{bmatrix} -3t + s/2 \\ 0 + s \\ t + 0 \end{bmatrix} = t\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}$$